An improved semi-analytical solution for determining water permeability of highly pervious porous materials

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Highlights

- Standard saturated water permeability tests are not applicable for highly pervious materials.
- For such materials, Stanic et al. (2023) have developed a new experimental procedure.
- Here is presented an improved semi-analytical solution for interpreting these experimental results.

Introduction

Implementation of highly pervious porous pavements for parking lots and roads, which main purpose is reduction of the stormwater surface runoff and protection from the pollution (Brite/Euram Report, 1994; Sambito et al., 2021; etc.), is one of most widely applied measures in urban catchments. Hence, determining water permeability of these materials is an obvious first step for modelling the effects of runoff reduction. In practice, two conventional experimental methods for laboratory investigation of water permeability of saturated porous materials are commonly used: the constant head permeability test and the falling head permeability test - Dirksen (1999), Das (2002), Reynolds et al. (2002), NEN5789 (2005). The first approach faces certain technical issues when performing on highly pervious materials since significant amount of water needs to be delivered to maintain constant water head, and hence the constant water flow through the sample, while the second one is not suitable because it neglects the Forchheimer's high-velocity flow (Forchheimer, 1901) through the sample and the influence of inertia on the fluid mass. To overcome these deficiencies Stanic et al. (2023) have proposed a new two-reservoirs test where water flows from one reservoir to another through the porous sample and the interconnecting pipe that create the flow resistance and hence damp the water level oscillations in the reservoirs. In the same study authors have derived semi-analytical solution describing this phenomenon, which accounts for the acceleration of the fluid mass (inertia) and the high-velocity flow through the laboratory setup and the porous sample. This solution has shown good accuracy when using fine time discretization, but also significant mitigation of results when using larger calculation time steps. Therefore, here is proposed an improved semi-analytical solution that is less sensitive to time step selection, and hence more robust and convenient for application. This new solution has been firstly compared with the experimental data of the two-reservoirs test given in Stanic et al. (2023), and then it has been tested against the existing semi-analytical solution for different calculation time steps.

Methodology

Two-reservoir test

Figure 1 illustrates the two-reservoir test based on the initially imposed water level difference between the two interconnected reservoirs. Water flows from one side to another (right to left in Figure 1) passing through the porous sample of thickness ΔL [L] and cross-sectional area A_s [L^2]. If the sample is highly permeable, the water levels in two reservoirs will not meet at equilibrium immediately but after damped oscillations driven by the conservation of the momentum.

The momentum equation describing this phenomenon relies on Newton's Second Law defining the change in momentum per change in time (I_t) as a difference between the water pressure and gravity force governing the flow (H_t) and the resistance force consisting of the sample impedance ($\Delta H_{imp,t}$) and the pipe friction ($\Delta H_{f,t}$) that oppose the flow. This equation is in length units, as forces are per weight and area unit:

$$H_t - \left(\Delta H_{f,t} + \Delta H_{imp,t}\right) = I_t \quad (1)$$

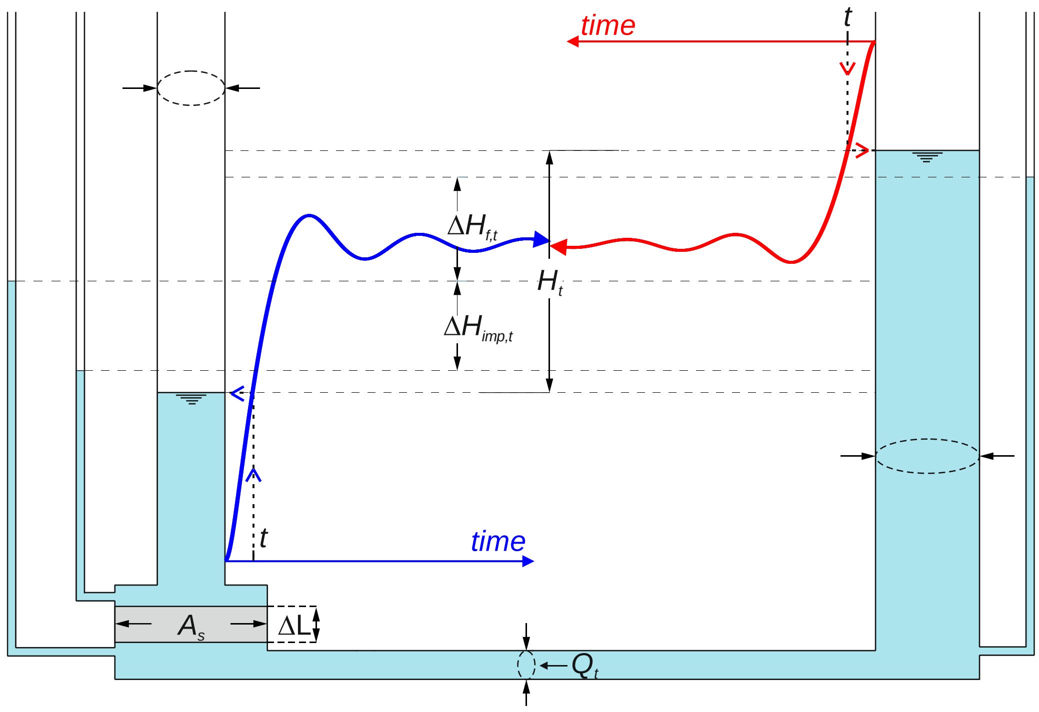

Figure 1. Schematic of the two-reservoirs test

By taking into account that: i) flow rate Q_t is proportional to $\frac{dH_t}{dt}$ according to the water balance equation, ii) $\Delta H_{imp,t}$ and $\Delta H_{f,t}$ are quadratic functions of Q_t according to Forchheimer (1901)'s law and Darcy-Weisbach equation (Nakayama & Boucher, 1998), respectively, and iii) I_t is a "fictive" inertial force proportional to $\frac{dQ_t}{dt} \sim \frac{d^2H_t}{dt^2}$, the differential form of Equation (1) is obtained:

$$-c_t A_e \frac{d^2H_t}{dt^2} - aA_e \frac{dH_t}{dt} - (r_Q + b)A_e{}^2 \frac{dH_t}{dt}\left|\frac{dH_t}{dt}\right| - H_t = 0 \tag{2}$$

where term $\frac{dH_t}{dt}\left|\frac{dH_t}{dt}\right|$ accounts for the flow direction change during oscillations, A_e is the equivalent cross-sectional area [L^2] depending on reservoirs geometry, c_t is a coefficient [T^2/L^2] depending on the pipe geometry, r_Q is the friction factor [T^2/L^5], while $a = \frac{\Delta L}{K_{sat}A_s}$ and $b = \frac{\beta \Delta L}{A_s{}^2}$ are coefficients depending on the two porous media parameters, the saturated hydraulic conductivity K_{sat} [L/T] and Forchheimer's coefficient β [T^2/L^2], respectively. The values of these two parameters are determined by fitting the solution of Equation (2) with experimental H_i data obtained from the two-reservoirs test.

New semi-analytical solution

Equation (2) has no continuous analytical solution over time t, but under certain assumptions it can be solved analytically at consecutive time intervals Δt. In Stanic et al. (2023) this is done by assuming constant pressure and gravity force H_t over short Δt, while here $\left|\frac{dH_t}{dt}\right|$ value is fixed during Δt to create more robust and physically based semi-analytical solution. If $\left|\frac{dH_t}{dt}\right|$ is constant Equation (2) reduces to the Ordinary Differential Equation of the second order which can be easily solved by means of the Laplace integral transform (Kreyszig et al. 2011). By shifting from time-domain to the so-called Laplace s-domain, differential Equation (2) transforms into the quadratic equation that can be easily solved and "translated" afterwards into the following H_t expression:

$$H_t = \frac{1}{r_1 - r_2}\left[H_{t-\Delta t}\left(r_1 e^{r_1 \Delta t} - r_2 e^{r_2 \Delta t}\right) + \left(fH_{t-\Delta t} - \frac{Q_{t-\Delta t}}{A_e}\right)\left(e^{r_1 \Delta t} - e^{r_2 \Delta t}\right)\right] \tag{3}$$

$$Q_t = -A_e \frac{dH_t}{dt} = -\frac{A_e}{r_1 - r_2}\left[H_{t-\Delta t}\left(r_1{}^2 e^{r_1 \Delta t} - r_2{}^2 e^{r_2 \Delta t}\right) + \left(fH_{t-\Delta t} - \frac{Q_{t-\Delta t}}{A_e}\right)\left(r_1 e^{r_1 \Delta t} - r_2 e^{r_2 \Delta t}\right)\right] \tag{4}$$

where index $(t - \Delta t)$ indicates values from the previous time step (initial condition), $f = \frac{a+(r_Q+b)|Q_t|}{c_t}$, while $r_1 = \frac{1}{2}\left(-f + \sqrt{f^2 - \frac{4}{c_t A_e}}\right)$ and $r_2 = \frac{1}{2}\left(-f - \sqrt{f^2 - \frac{4}{c_t A_e}}\right)$ are the roots of the quadratic equation in Laplace domain. Values of H_t and Q_t are determined for each time step by iteratively solving Equations (3) and (4) until the difference in two consecutive iterations is negligible. Then, newly obtained H_t and Q_t values are used as the initial condition for the next time step $(t + \Delta t)$. As in case of the solution presented in Stanic et al. (2023), this approach also reduces to the standard falling head permeability formula for low permeability materials.

Results and discussion

Comparison with experimental data from Stanic et al. (2023)

The two-reservoirs test has been conducted in the Hydraulic Lab of the Faculty of Civil Engineering University of Belgrade. Material that has been tested is highly pervious porous pavement used for parking lots and roads, which main purpose is reduction of the stormwater surface runoff, but also protection from the pollution (Brite/Euram Report, 1994; Sambito et al., 2021; etc.). Samples consisting of the expanded clay particles (1 – 4 mm), with size 20cm x 20cm x 6cm and porosity of 35%, have been used in Stanic et al. (2023) to conduct the experimental results that are used here to validate the proposed semi-analytical solution - Equation (3).

Figure 2 - left illustrates comparison between the experimental H_i data (empty dots) obtained by continuous monitoring of the water level difference in two reservoirs, and H_t data (solid line) calculated with Equation (3). The best agreement between the two (presented in Figure 2 - left) is obtained for K_{sat} = 1.3x10^{-2} m/s and β = 3.1x10^3 s^2/m^2, values almost identical to those in Stanic et al. (2023). Additionally, in Figure 2 – right values of Q_t calculated by means of Equation (4) are compared with the experimental Q_i data obtained from H_i measurements as $Q_i = \frac{H_{i+1} - H_{i-1}}{t_{i+1} - t_{i-1}} A_e$.

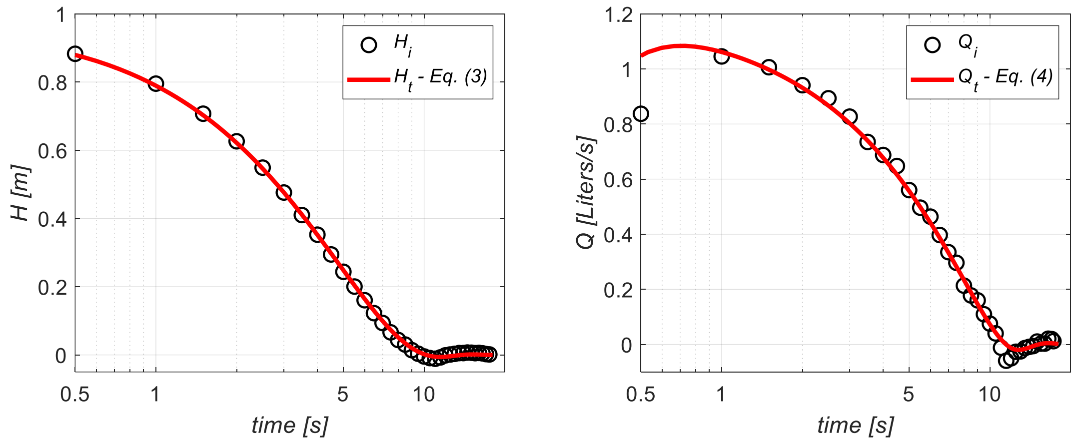

Figure 2. Comparison between the experimental two-reservoirs data from Stanic et al. (2023) and the presented semi-analytical approach: Left – experimental H_i data (empty dots) against Equation (3) (solid line); Right - experimental Q_i data (empty dots) against Equation (4) (solid line)

Comparison with semi-analytical solution from Stanic et al. (2023)

In Stanic et al. (2023) is presented semi-analytical solution of Equation (2) derived under the assumption the term H_t is constant over short Δt. As illustrated in Figure 3 – left, such approach provides accurate H_t values for $\Delta t \leq T/100$, where T is the period of oscillations [s], while results are significantly mitigated for larger Δt values. On the other hand, Equation (3) appears to be less sensitive to time discretization, showing rather similar H_t values for variety of Δt. Also, the solution proposed here is more robust and convenient for application since the change in flow direction is automatically accounted for by updating $|Q_t| \sim \left|\frac{dH_t}{dt}\right|$ value in iterations.

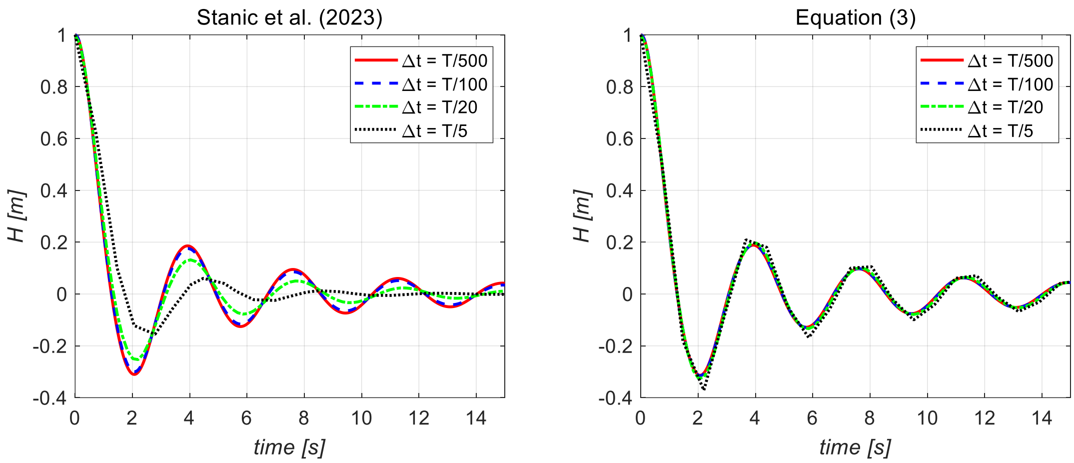

Figure 3. Left – Semi-analytical solution from Stanic et al. (2023) for different Δt values; Right – Equation (3) for different Δt values (T = 3.66 s). Calculations presented in this Figure are performed on hypothetical example of highly permeable material with K_{sat} = 0.5 m/s and β = 200 s^2/m^2 to illustrate higher amplitudes of oscillations

Conclusions and future work

An improved semi-analytical solution proposed in this work shows rather good agreement with the two-reservoirs experimental data, but also clear advantages over the solution in Stanic et al. (2023). Firstly, the improved solution is less sensitive to time discretization, which is why it is more robust and better for practical application. Also, the calculation algorithm is simpler since the change in flow direction is automatically accounted for. As with the previous solution this one can also be reduced to the standard falling-head permeability formula and used for less permeable materials. Hence, to improve this approach additionally future work should focus more on the mathematical properties of Laplace transform that can be used to estimate the occurrence of the water level oscillations prior running the calculation. That way, this semi-analytical solution can be additionally simplified, and iterative algorithm can be avoided when not necessary.

Acknowledgment

This research was funded by the Science Fund of the Republic of Serbia through the IDEJE national project Ø-Waste-Water (No 7737365) and by the European Union-funded project euPOLIS under the Horizon 2020 program H2020-EU.3.5.2 (No 869448). Also, this is a preparation part of the ongoing project “Road and Environmental Safety related to water – pavement, Interactions” (RESAFE), No. 7051, from Program PRISMA, financed by Science Fund of the Republic of Serbia.

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